LA-UR-18-24204

Title: Variance Reduction Introduction & Overview

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Intended for: Attila4MC Class

Issued: 2018-05-14

Variance Reduction

Introduction & Overview

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XCP-3: Monte Carlo Codes, Methods and Applications

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This Section's Objective

- **Introduce the user to <u>some</u> of MCNP's long standing importance based variance reduction techniques**

Variance Reduction: Introduction

Out of the box, MCNP can be a little slow for certain types of problems.

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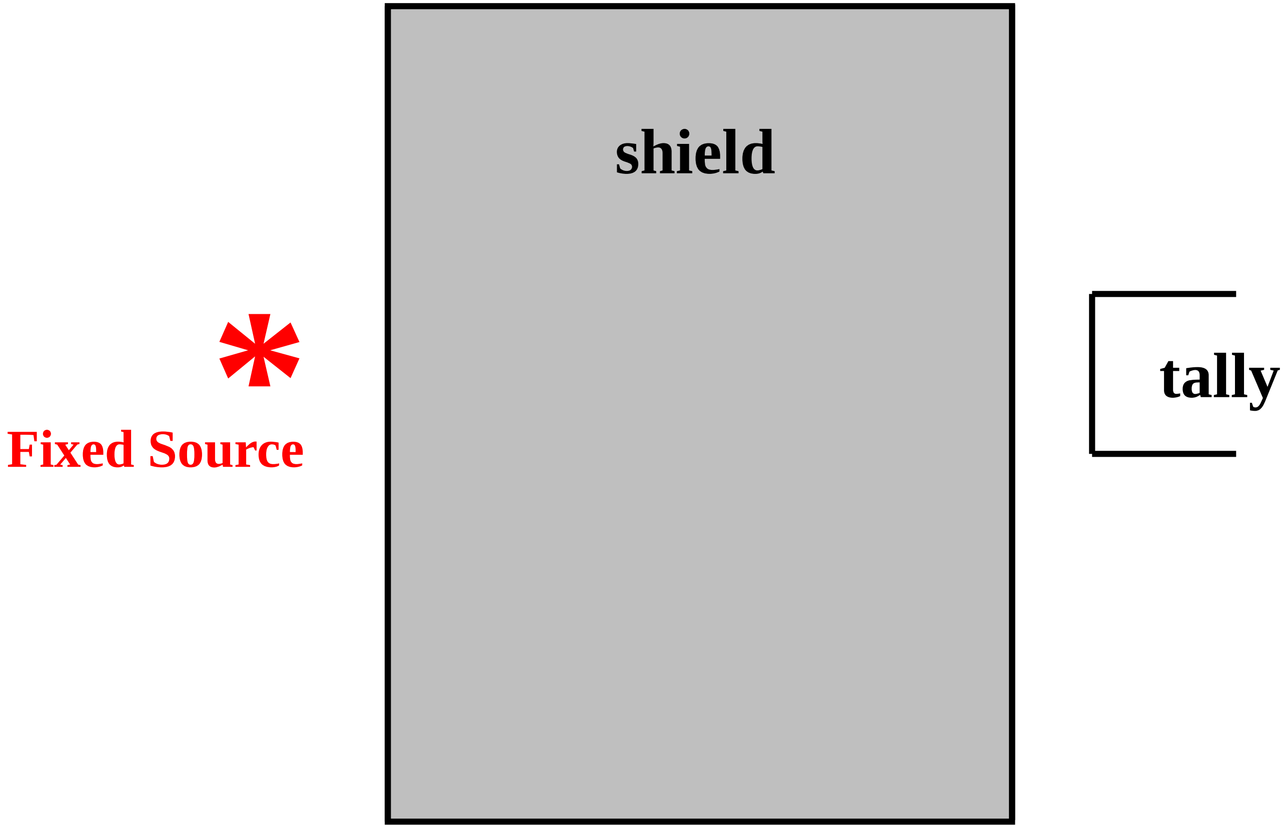
How To Speed Up Fixed Source Problems



GOAL

Reduce the computer time required to obtain results of sufficient precision.

MCNP with variance reduction

MOTIVATION

- **Exchange user time for computational time**
- **A few hours of user time often reduces computational time by factors of 10 to 1000**

How do we do this?

Given a tally, preferentially sample "important" random walks that contribute to the tally at the expense of "unimportant" random walks.

ANALOG MONTE CARLO

Sample events according to their natural physical probabilities.

NON-ANALOG or BIASED MONTE CARLO

Do not directly simulate nature's laws.

Bias the PDF's in a fair way to produce a more efficient random walk.

Biased Monte Carlo

Numerous techniques to use in MCNP

- Simple methods are the focus of this course
- Advanced methods are the focus of the follow-on course(s)

Strategy for this class

- Introduce a variance reduction concept
- Try it in the sample problem
- Study the results
- Discuss any user concerns
- Repeat the process and show that for the most part these techniques can be used together

IMPLEMENTATION STRATEGY

- **Can be more art than science.**
- **Each technique has its own advantages, problems, and peculiarities.**
- **For <u>success</u>, the user must understand both the physical problem and the variance reduction techniques.**

SELECT A TECHNIQUE

Supplying appropriate parameters may be more difficult than selection of the technique.

Better to err on the side of too little biasing rather than too much biasing.

Rely on:

- Experience with similar problems
- Make a series of short exploratory runs

Examine Exploratory Run Output

- The techniques are improving the sampling of the tally particles
- The techniques are working cooperatively
- The Tally Fluctuation Chart (TFC) is not erratic
- Nothing looks obviously ridiculous

Erratic Error Estimates

Generally, MCNP tallies are finite variance tallies.

If the 10 statistical checks are not satisfied, the error estimates should be considered erratic and unreliable.

```
1status of the statistical checks used to form confidence intervals for the mean for each tally bin


 tally   result of statistical checks for the tfc bin (the first check not passed is listed) and error magnitude check for all bins

     1   passed the 10 statistical checks for the tally fluctuation chart bin result
         passed all bin error check:     1 tally bins all have relative errors less than 0.10 with no zero bins
```

```
1status of the statistical checks used to form confidence intervals for the mean for each tally bin


 tally   result of statistical checks for the tfc bin (the first check not passed is listed) and error magnitude check for all bins

     1   missed  7 of 10 tfc bin checks: the relative error exceeds the recommended value of 0.1 for nonpoint detector tallies
         missed all bin error check:     1 tally bins had     0 bins with zeros and     1 bins with relative errors exceeding 0.10
```

Your 1st clue that something may be wrong!

Erratic Error Estimates (cont.)

- **Typically occur because a high-weight particle tallies from an important region of phase-space that has not been well sampled.**
- **Ensure that phase-space regions are well sampled by many particles and try to minimize the weight fluctuations among these particles.**
  - Use biasing techniques that preferentially push particles into important regions without introducing large weight fluctuations.
- **Weight windows can be very useful in minimizing weight fluctuations caused by other biasing techniques.**

Some More Definitions

- **Weight** – A number representing a particle's relative contribution to a tally.
- **Importance** – Proportional (in some manner) to the expected score generated by a unit weight particle. Often this definition is associated with an adjoint solution of a transport equation.
- **Cell Importance** -- Relative values associated with cells that are needed in creating ratios used in geometry splitting and Russian roulette.

Definitions

True Mean:

$$E(x) = \int_{-\infty}^{\infty} x \, \mathbf{f}(\mathbf{x}) \, \mathbf{dx}$$

Estimated Mean:

$$X_n = \frac{1}{n}\sum_{i=1}^{n} x_i$$

True Variance:

$$\sigma^2 = E\left(x^2\right) - E^2(x)$$

Estimated Sample Variance:

$$S^2 = \frac{n}{n-1}\left(\left[\frac{1}{n}\sum_{i=1}^{n} x_i^2\right] - X_n^2\right)$$

Definitions

True Variance of the Mean: $$\sigma_m^2 = \frac{\sigma^2}{n}$$

Estimated Variance of the Mean: $$S_m^2 = \frac{S^2}{n}$$

Estimated Relative Error: $$R = \frac{S_m}{X_n}$$

Variance Reduction Principles

<u>Efficiency</u>: a measure of how quickly the desired precision is achieved.

- Monte Carlo codes can estimate this (figure of merit):

$$FOM = \frac{1}{R^2 T} \qquad R^2 \propto \frac{1}{n} \qquad T \propto n$$

FOM should be constant with n
Larger FOM more efficient

- Factors affecting efficiency:

History-scoring efficiency
Dispersion in non-zero history scores
Computer time per history

Most techniques decrease R^2 more than T is increased.

$$FOM = \frac{1}{R^2 T}$$

Types of Variance Reduction Techniques

Truncation method – truncates parts of phase-space that do not contribute significantly to the solution.

- geometry truncation
- energy cutoff
- time cutoff

Types of Variance Reduction Techniques

Population control method – uses particle splitting and Russian roulette to control the number of samples taken in various regions of phase-space.

- geometry splitting & Russian roulette
- energy splitting & Russian roulette
- time splitting & Russian roulette
- weight cutoff
- weight windows

Types of Variance Reduction Techniques

Modified sampling method – alters the statistical sampling of a problem to increase the number of tallies for a fixed computer time.

- exponential transform
- source biasing
- neutron-induced photon production biasing

Types of Variance Reduction Techniques

Partially or fully deterministic methods – circumvent part of the normal Monte Carlo process (random walk + tally) by using known expected values.

- implicit capture (absorption by weight reduction)
- forced collisions
- point / ring detectors
- DXTRAN
- correlated sampling

This Class:

- **This Lecture**
  - Implicit capture
  - Splitting & Russian roulette
- **Following Lecture**
  - Weight windows
  - Stochastic and Deterministic Weight window generators
  - Consistent Adjoint Driven Importance Sampeling (CADIS)
  - Forward Weighted CADIS Method

Advanced Classes:

The other techniques

Exercise Objective

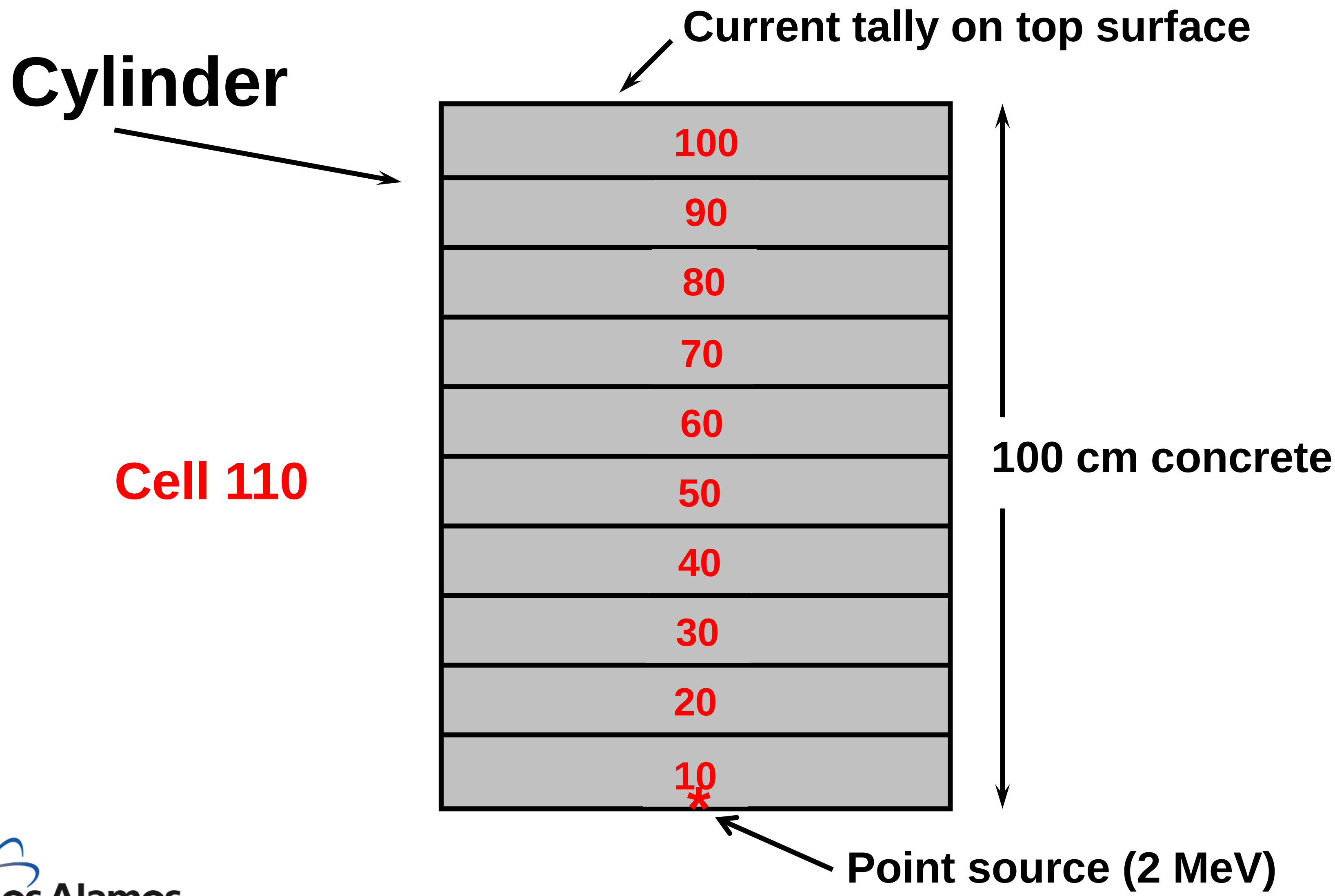
- **The problem is described on the next few slides.**
- **Our objective in this session is to try various variance reduction techniques and tune them with reasonable effort to achieve as an efficient calculation as possible.**
- **We will use Attila4MC to automatically set importance values in the next session**
  - NOTE: We will be adding the techniques one at a time and using them together.

General Guidance When Using VR

For each exercise, complete the appropriate worksheet table entry from:

- **Examine "print table 126". Look at "tracks entering" each cell and the total.**
- **Review the statistical checks.**
- **Review the tally fluctuation chart(s).**
- **Record the "computer time".**

Problem Description



Problem Input - var0

Class variance reduction problem

```
  Revised class variance reduction problem #1: var0
c
c  Analog capture
c
 10  100  -2.03  -1   -3   2
 20  100  -2.03  -1   -4   3
 30  100  -2.03  -1   -5   4
 40  100  -2.03  -1   -6   5
 50  100  -2.03  -1   -7   6
 60  100  -2.03  -1   -8   7
 70  100  -2.03  -1   -9   8
 80  100  -2.03  -1  -10   9
 90  100  -2.03  -1  -11  10
100  100  -2.03  -1  -12  11
110   0          1:-2:12

1   cy  100
2   py  0
3   py  10
4   py  20
5   py  30
6   py  40
7   py  50
8   py  60
9   py  70
10  py  80
11  py  90
12  py  100

c  the following is pseudo-concrete
m100  1001 -0.02  8016 -0.60  14000 -0.38
sdef  pos= 0. 1.e-6  0.  erg=2
c
imp:n  1  9R  0
c
f1:n 12
nps 100000
cut:n 2j 0
print
```

The Cutoffs Input Card

cut:<pl>   T   E   WC1   WC2   SWTM

<pl>	=	N (neutrons), P (photons), E (electrons)
T	=	time cutoff in shakes, 1 shake = 10^{-8} sec
E	=	lower energy cutoff in MeV
WC1	=	upper weight cutoff
WC2	=	lower weight cutoff
SWTM	=	minimum source weight

Variance Reduction Exercise #0

Analog Calculation

- Make a copy of the file `var0` or `var0.txt` with an appropriate file name
- Add the following card for analog capture:

```
cut:n 2j 0
```

- Ensure the imp and nps cards are:

```
imp:n  1  9R  0
nps  100000
```

- Include your rand card:

```
rand  seed= ??
```

- Run the calculation in sequential mode (no threads)

`mcnp6 n=var0` or `mcnp6 n=var0.txt`

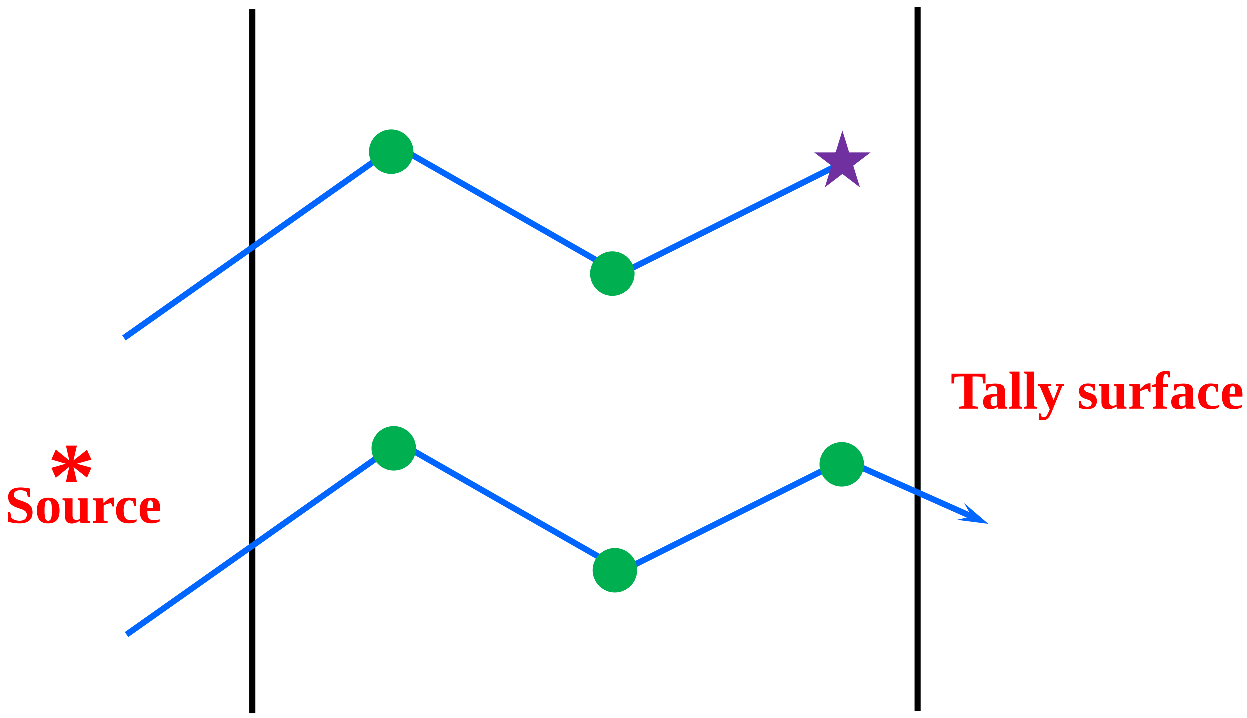
Exercise #0 Selected Output

```
1neutron  activity in each cell                                                                          print table 126

                 tracks     population   collisions   collisions     number        flux        average      average
        cell    entering                               * weight     weighted     weighted    track weight  track mfp
                                                    (per history)    energy       energy     (relative)      (cm)

     1    10     129360      100000       909327     9.0933E+00   3.3803E-04   6.9053E-01   1.0000E+00   2.6644E+00
     2    20      55597       20081       790019     7.9002E+00   5.0106E-05   1.7602E-01   1.0000E+00   1.5834E+00
     3    30      25947        9241       384917     3.8492E+00   1.7676E-05   7.2653E-02   1.0000E+00   1.3332E+00
     4    40       9647        3467       144105     1.4411E+00   8.8648E-06   3.9330E-02   1.0000E+00   1.2395E+00
     5    50       3042        1144        46451     4.6451E-01   4.3808E-06   2.0964E-02   1.0000E+00   1.1838E+00
     6    60        969         352        14439     1.4439E-01   3.9019E-06   1.9514E-02   1.0000E+00   1.1755E+00
     7    70        286         116         4115     4.1150E-02   2.9942E-06   1.5653E-02   1.0000E+00   1.1707E+00
     8    80         57          23         1022     1.0220E-02   3.6870E-08   5.6521E-08   1.0000E+00   1.1060E+00
     9    90         16           7          220     2.2000E-03   3.8440E-08   5.0022E-08   1.0000E+00   1.1206E+00
    10   100          0           0            0     0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00   0.0000E+00

     total       224921      134431      2294615     2.2946E+01



1status of the statistical checks used to form confidence intervals for the mean for each tally bin


 tally   result of statistical checks for the tfc bin (the first check not passed is listed) and error magnitude check
     for all bins

     1   no nonzero tallies were made in the tally fluctuation chart bin
         no nonzero tallies were made in any one of the      1 tally bins
```

Exercise #0 Selected Output

```
1tally fluctuation charts

                      tally    1
          nps        mean      error    vov  slope    fom
         8000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        16000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        24000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        32000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        40000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        48000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        56000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        64000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        72000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        80000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        88000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
        96000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00
       100000   0.0000E+00 0.0000 0.0000   0.0 0.0E+00

***********************************************************************************************************************

dump no.    2 on file var0r     nps =      100000     coll =        2294615     ctm =        0.14    nrn =     38883473
```

Analog vs. Implicit Capture



Implicit Capture – a deterministic splitting process where the particle is split into an absorbed weight (which is not tracked) and a surviving weight.

Applied at each collision. (The particle always survives the collision.)

Surviving weight given by: $$W \times \left(1 - \frac{\sigma_{a_i}}{\sigma_{T_i}}\right)$$

σ_{a_i} = collision nuclide i absorption cross section

σ_{T_i} = collision nuclide i total cross section

Automatically "on" – see WC1 parameter on the CUT card and PHYS:N card.

Particles can reach very low weights (will discuss solution later).

Variance Reduction Exercise #1

Implicit Capture

- Make a copy of the input file and call it var1.
- Remove the `cut` card.
- Keep the `nps` card with 100,000 particles:

```
nps  100000
```

- Keep your rand card:

```
rand  seed= ??
```

- Run the calculation.

  `mcnp6 n=var1` or `mcnp6 n=var0.txt`

- Complete the worksheet.
- How do the results compare to the analog calculation?

Exercise #1 Selected Output

```
1neutron  activity in each cell                                                                                 print table 126

                  tracks     population   collisions   collisions     number        flux        average      average
        cell      entering                              * weight      weighted      weighted    track weight  track mfp
                                                       (per history)  energy        energy      (relative)     (cm)

    1     10      143213       100000      1243828    9.1553E+00   3.3412E-04   6.8783E-01   8.2312E-01   2.6585E+00
    2     20       81001        20704      1324079    8.0285E+00   4.9327E-05   1.7439E-01   6.4840E-01   1.5794E+00
    3     30       45857        11848       746842    3.8802E+00   1.8161E-05   7.4351E-02   5.4296E-01   1.3388E+00
    4     40       19817         5614       320711    1.4682E+00   9.0309E-06   3.9499E-02   4.7148E-01   1.2419E+00
    5     50        7264         2291       112809    4.7314E-01   4.5570E-06   2.0358E-02   4.2784E-01   1.1847E+00
    6     60        2286          784        35497    1.3955E-01   2.6649E-06   1.2360E-02   3.9726E-01   1.1559E+00
    7     70         675          235        10269    3.9315E-02   1.8176E-06   6.9485E-03   3.8651E-01   1.1505E+00
    8     80         173           60         3590    1.3050E-02   3.8749E-08   6.2217E-08   3.6175E-01   1.1144E+00
    9     90          37           15          356    1.2016E-03   3.8300E-08   4.8750E-08   3.3832E-01   1.1208E+00
   10    100           7            2           89    3.3196E-04   2.6075E-08   3.5759E-08   3.8344E-01   1.0374E+00

     total        300330       141553      3798070    2.3199E+01
```

```
1status of the statistical checks used to form confidence intervals for the mean for each tally bin

 tally   result of statistical checks for the tfc bin (the first check not passed is listed) and error magnitude check for all bins

     1   missed  7 of 10 tfc bin checks: the relative error exceeds the recommended value of 0.1 for nonpoint detector tallies
         missed all bin error check:     1 tally bins had     0 bins with zeros and     1 bins with relative errors exceeding 0.10
```

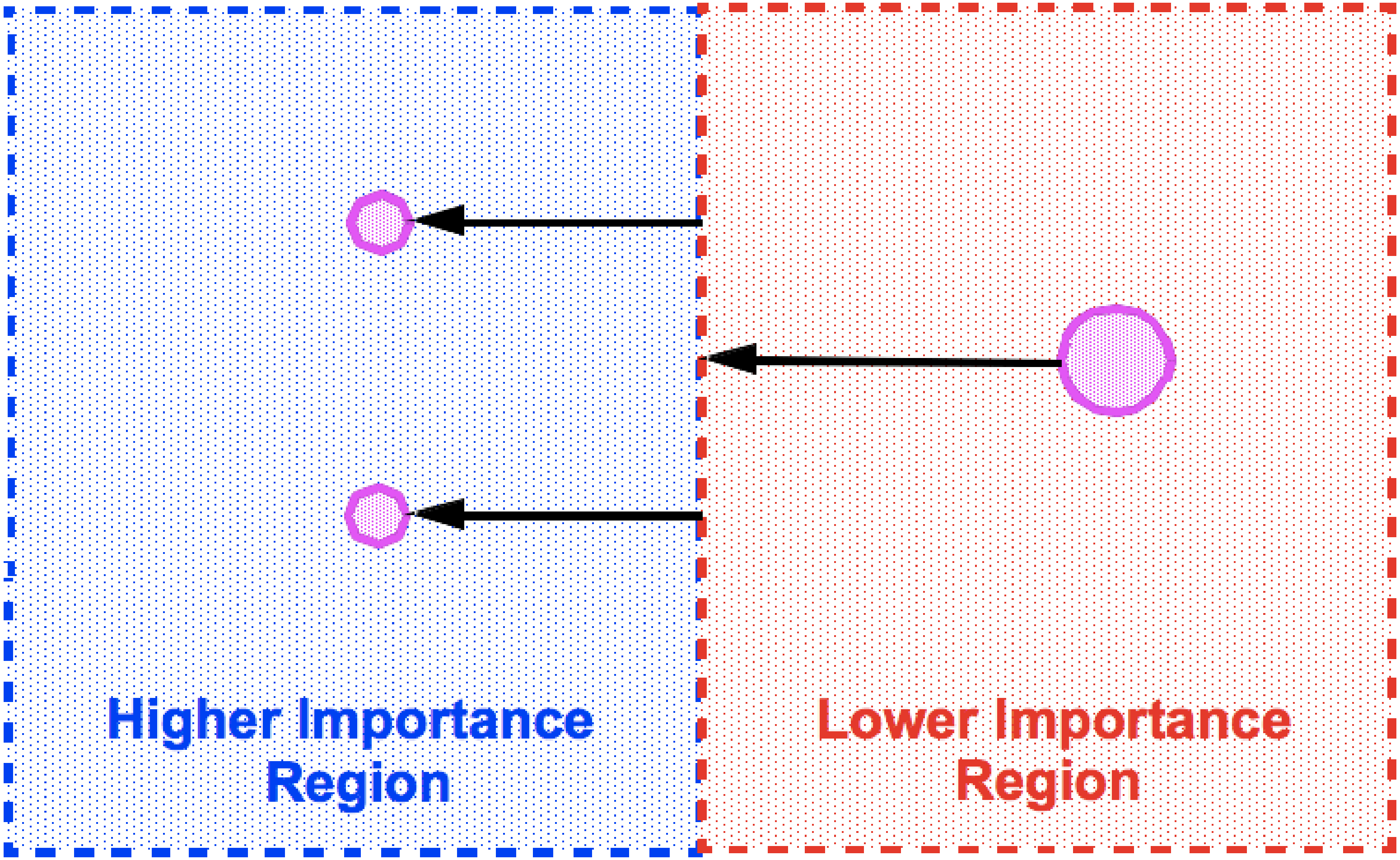
Exercise #1 Selected Output

```
1tally fluctuation charts

                      tally    1
         nps        mean     error    vov  slope    fom
        8000   0.0000E+00 0.0000 0.0000  0.0 0.0E+00
       16000   0.0000E+00 0.0000 0.0000  0.0 0.0E+00
       24000   0.0000E+00 0.0000 0.0000  0.0 0.0E+00
       32000   0.0000E+00 0.0000 0.0000  0.0 0.0E+00
       40000   0.0000E+00 0.0000 0.0000  0.0 0.0E+00
       48000   6.4271E-06 1.0000 0.9999  0.0 8.9E+00
       56000   5.5090E-06 1.0000 0.9999  0.0 7.7E+00
       64000   4.8203E-06 1.0000 1.0000  0.0 6.7E+00
       72000   4.2848E-06 1.0000 1.0000  0.0 6.0E+00
       80000   3.8563E-06 1.0000 1.0000  0.0 5.4E+00
       88000   3.5057E-06 1.0000 1.0000  0.0 4.9E+00
       96000   3.2136E-06 1.0000 1.0000  0.0 4.5E+00
      100000   3.0850E-06 1.0000 1.0000  0.0 4.3E+00

 ***********************************************************************************************************************

 dump no.    2 on file var1r     nps =      100000     coll =        3798070     ctm =         0.23    nrn =   62798675
```

Geometry Splitting
and
Russian Roulette

Variance Reduction Techniques

Splitting



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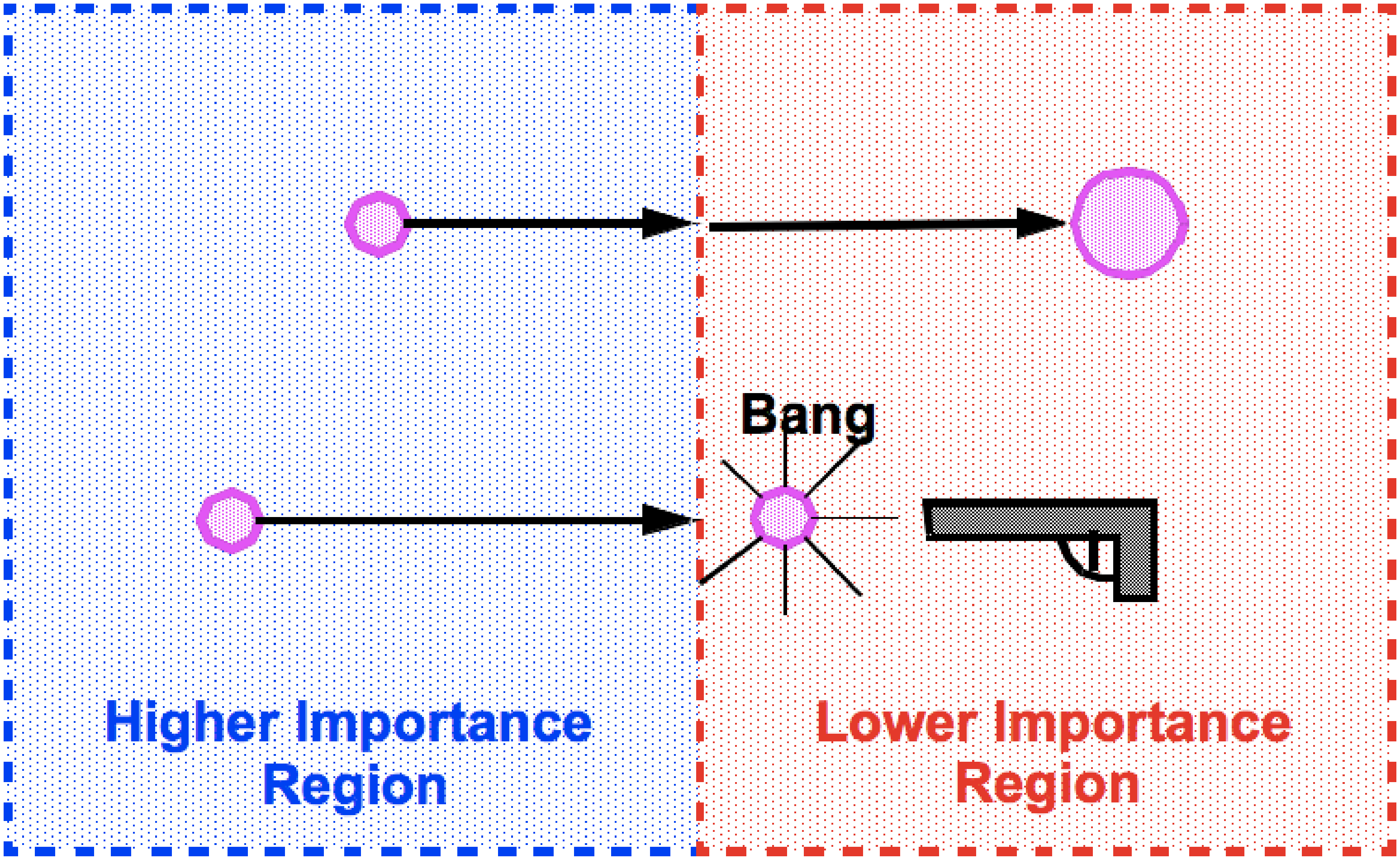
Russian Roulette



Geometry Splitting and Russian Roulette

Particles in more important regions are increased in number for better sampling and decreased in number in less important regions.

Applied at surface crossing using IMP card entries (user provided).

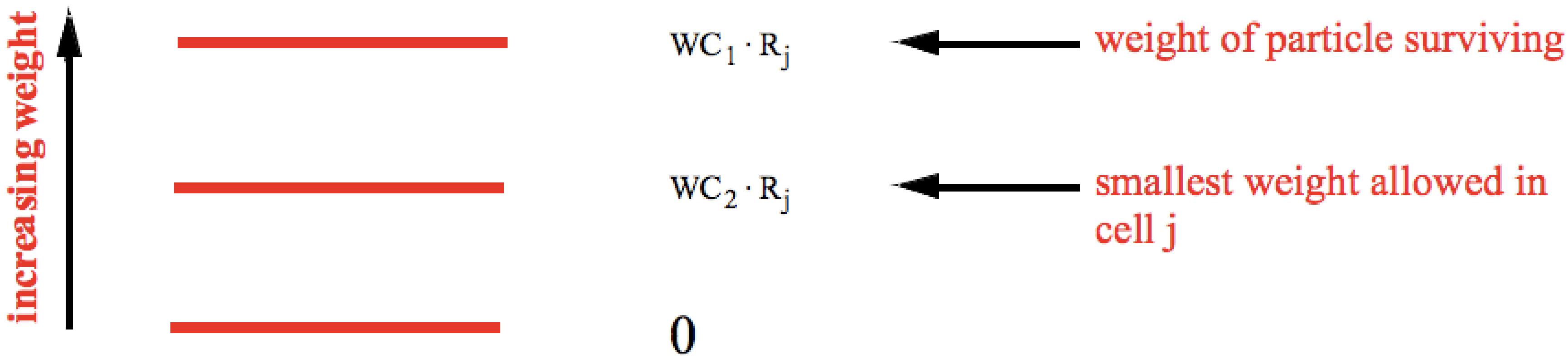
If the cell IMP ratio (new cell to previous cell) `v > 1`, the particle is split into `v` particles with weight `W/v`. (See Chapter 2 of the MCNP5 manual for non-integer `v`.)

If the ratio is less (`v < 1`) than one, Russian roulette is played.

Kill probability: `1 – v` Survival probability: `v` Survival weight: `W/v`

Weight Cutoff

Russian roulette is played if a particle's weight drops below a user-specified weight cutoff. The particle is either killed or survives with increased weight. This is <u>often</u> an implicit capture induced problem.



R_j = ratio of (source cell) / (cell_j) importances

Designed to solve problems produced by implicit capture event. Without weight cutoff, particle weight would become too small.

Weight Cutoff (cont.)

Unlike other cutoffs, weight cutoff is NOT biased. It is a roulette.

Applied when $W < R_j \times WC2$; the surviving particle has:

survival probability	$W / (WC1 \times R_j)$
new weight	$WC1 \times R_j$

(R_j = ratio of source cell / cell j importances)

Automatically "on" – see `WC1` and `WC2` parameters of `CUT` card.

Geometry Splitting

- Invoke with the `IMP` card (no defaults)

  `IMP:n` I_1 I_2 I_3 … (I_j = cell j importance)

- IMP card required for at least one particle type
- Typically $I_j = 1$ for source cell(s)
- Keep neighbor cell ratios $(I_j / I_k) < 8$
- Keep the number of tracks constant from the source to the tally region

Geometry Splitting and Russian Roulette

- **Copy your input file from Exercise #1. Call it var2a.**
- **Try your "best guess" at the cell importance (imp) values. (HINT: Look at how the tracks entering or population is varying from cell to cell in print table 126.)**
- **Use the `imp:n` card to alter the slab's population distribution with the goal being to maximize the FOM for Tally 1.**
- **Consider using the "`m`" notation: `imp:n 1 2m 3m…`**

  (m means multiply previous entry by this value. Can't be used with r notation.)
- **Run the problem and complete the worksheet.**

- **STOP! Do not pass GO. Do not collect $200.**

Exercise #2A Selected Output

```
1neutron  activity in each cell                                                                              print table 126

               tracks      population   collisions   collisions     number       flux        average       average
        cell   entering                              * weight      weighted     weighted   track weight   track mfp
                                                    (per history)   energy       energy     (relative)      (cm)

    1     10     143220       123083      1238292    9.1449E+00  3.3629E-04  6.8846E-01  8.2517E-01  2.6603E+00
    2     20     162019       102999      2631442    7.9978E+00  4.9317E-05  1.7407E-01  6.4987E-01  1.5792E+00
    3     30     184738       118252      3041341    3.9346E+00  1.7696E-05  7.3287E-02  5.4011E-01  1.3337E+00
    4     40     161710       105696      2605009    1.4862E+00  8.3116E-06  3.6376E-02  4.6898E-01  1.2333E+00
    5     50     118501        78913      1867703    4.8621E-01  4.4814E-06  2.0070E-02  4.2379E-01  1.1843E+00
    6     60      77057        51994      1197899    1.4712E-01  2.5122E-06  1.1101E-02  3.9729E-01  1.1569E+00
    7     70      48338        32696       736213    4.3455E-02  1.5143E-06  6.8465E-03  3.8045E-01  1.1419E+00
    8     80      28504        19387       436927    1.2537E-02  9.7604E-07  4.1966E-03  3.6858E-01  1.1326E+00
    9     90      15674        10770       238546    3.3367E-03  5.1778E-07  2.2043E-03  3.5852E-01  1.1241E+00
   10    100       7144         5267       108583    7.5301E-04  5.6996E-07  2.3705E-03  3.5586E-01  1.1249E+00

    total        946905       649057     14101955    2.3257E+01



1status of the statistical checks used to form confidence intervals for the mean for each tally bin


 tally   result of statistical checks for the tfc bin (the first check not passed is listed) and error magnitude check for all bins

    1    missed  2 of 10 tfc bin checks: the estimated mean has a trend during the last half of the problem
         passed all bin error check:     1 tally bins all have relative errors less than 0.10 with no zero bins
```

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Exercise #2A Selected Output

```
1tally fluctuation charts

                      tally      1
         nps         mean      error    vov   slope     fom
        8000   6.0536E-06 0.2004 0.0986  0.0       374
       16000   4.9501E-06 0.1431 0.0571  0.0       371
       24000   5.6025E-06 0.1075 0.0311  0.0       434
       32000   5.8500E-06 0.0877 0.0206  0.0       476
       40000   6.0954E-06 0.0797 0.0173  0.0       457
       48000   6.5589E-06 0.0794 0.0399  0.0       384
       56000   6.4217E-06 0.0743 0.0331  0.0       379
       64000   6.3404E-06 0.0696 0.0274  0.0       378
       72000   6.1814E-06 0.0669 0.0234  0.0       363
       80000   6.1655E-06 0.0649 0.0241  0.0       346
       88000   6.1081E-06 0.0623 0.0211  0.0       343
       96000   6.1024E-06 0.0594 0.0185  0.0       345
      100000   6.0601E-06 0.0588 0.0174  0.0       339
```

Geometry Splitting Ratio Method

	Tracks	IMP	New IMP
Source	300	1	1
	200	2	3 = (2/1) x (300/200) x 1
	100	4	12 = (4/2) x (200/100) x 3
Tally	25	8	96 = (8/4) x (100/25) x 12

The vrtsplit1.pl (PERL) script makes this easy.

```
This script (vrtsplit1.pl) adjusts cell importances to
obtain a constant track distribution and requires 1 or 2
input files on the command line.

The first file to appear on the command line must be an
MCNP output file (outp) with Print Tables 60 & 126.

The second file is optional.  This file contains cell numbers,
separated by white space, for which the ratios are calculated.
Cell numbers may be all on one line, one per line, or a mixture.

If the second file is not present, the script will calculate cell
importances and ratios for all cells present in Print Table 60
that have non-zero tracks and non-zero importances.

If the second file is present, the cell importances are calculated
for only those cells present in the file for all that have non-
zero tracks and non-zero importances.  Ratios are calculated
for cell pairs starting from the first pair of cell numbers.
The first two cell numbers form the first pair.  The second and
third cell numbers form the second pair, etc.

Cell importances appear in the default or specified order in
the file: importance.inp

Cell importance ratios appear in m-notation in the default
or specified order in the file: splitratio.inp
```

Geometry Splitting and Russian Roulette

- **Copy your input file from Exercise #2A. Call it var2b.**
- **Now use the "ratio" method to determine new values for the imp card (i.e., use vrtsplit1.pl).**
- **If the population is not ~constant through the slab, make another "adjustment" to the `imp` card values. (This may be needed in more complex geometries.)**
- **Consider using the "`m`" notation: `imp:n 1   2m   3m …`**

  (m means multiply previous entry by this value. Can't be used with r notation.)
- **Run the problem and complete the worksheet.**

Exercise #2B Selected Output

```
1neutron  activity in each cell                                                                  print table 126

                tracks     population   collisions   collisions     number        flux        average      average
        cell   entering                              * weight      weighted     weighted   track weight  track mfp
                                                   (per history)    energy       energy     (relative)      (cm)

     1    10     143046      120256      1242442    9.1713E+00   3.3461E-04   6.8794E-01   8.2467E-01   2.6592E+00
     2    20     142146       83619      2313725    7.9539E+00   5.0053E-05   1.7522E-01   6.5121E-01   1.5827E+00
     3    30     140335       87610      2300974    3.8626E+00   1.8138E-05   7.4974E-02   5.4318E-01   1.3384E+00
     4    40     140233       99990      2255418    1.4626E+00   8.6381E-06   3.8011E-02   4.7108E-01   1.2367E+00
     5    50     141409      107814      2220064    4.8096E-01   4.8088E-06   2.1632E-02   4.2552E-01   1.1886E+00
     6    60     145249      114755      2242872    1.4843E-01   2.5462E-06   1.1695E-02   3.9756E-01   1.1573E+00
     7    70     144471      115653      2220645    4.4091E-02   1.5467E-06   6.8583E-03   3.7885E-01   1.1415E+00
     8    80     143312      117209      2158850    1.2277E-02   8.5087E-07   3.6270E-03   3.6670E-01   1.1312E+00
     9    90     146124      121924      2192008    3.3549E-03   4.2482E-07   1.7310E-03   3.5859E-01   1.1243E+00
    10   100     144277      126550      2166336    7.4930E-04   2.7492E-07   9.7280E-04   3.5547E-01   1.1220E+00

     total      1430602     1095380     21313334    2.3140E+01


1status of the statistical checks used to form confidence intervals for the mean for each tally bin


 tally   result of statistical checks for the tfc bin (the first check not passed is listed) and error magnitude check for all bins

     1   passed the 10 statistical checks for the tally fluctuation chart bin result
         passed all bin error check:     1 tally bins all have relative errors less than 0.10 with no zero bins
```

Exercise #2B Selected Output

```
1tally fluctuation charts

                        tally    1
       nps         mean     error   vov  slope     fom
      8000   5.2856E-06 0.1202 0.0386  0.0     700
     16000   5.8969E-06 0.0927 0.0434  0.0     568
     24000   6.3918E-06 0.0705 0.0236  0.0     637
     32000   6.0776E-06 0.0615 0.0186  6.4     641
     40000   6.0829E-06 0.0540 0.0141  7.2     659
     48000   6.1609E-06 0.0503 0.0142  7.2     631
     56000   6.2308E-06 0.0459 0.0114  8.1     646
     64000   6.3166E-06 0.0432 0.0097 10.0     636
     72000   6.1657E-06 0.0405 0.0088 10.0     649
     80000   6.2678E-06 0.0384 0.0079 10.0     649
     88000   6.2309E-06 0.0366 0.0071 10.0     652
     96000   6.1994E-06 0.0351 0.0066 10.0     652
    100000   6.1676E-06 0.0344 0.0063  6.6     651
```